Trinity College
HARTFORD CONNECTICUT

Department of Economics
Trinity College
Hartford, CT
06106 USA

<http://www.trincoll.edu/depts/econ/>

**TRINITY COLLEGE DEPARTMENT OF ECONOMICS
WORKING PAPER 09-01**

**Neoclassical Growth Theory and Heterodox Growth Theory:
Opportunities For and Obstacles To Greater Engagement**

Mark Setterfield

December 2009

Abstract

This paper explores the possibilities for and likely impediments to greater engagement between neoclassical and heterodox growth theorists. Simple structural models are used to identify the essential “mechanics” of the growth process in both the neoclassical and heterodox traditions, and these are shown to point to important areas of theoretical overlap and even observational equivalence. It is argued, however, that the resultant opportunities for greater engagement between growth theorists are tempered by a number of obstacles, that are methodological, rhetorical and sociological in nature.

J.E.L. classification codes: O40, E12, E13

Keywords: Neoclassical growth theory, heterodox growth theory, endogenous growth

1. Introduction

Growth theory has a long and illustrious history in economics, and has occupied some of the disciplines great minds.¹ It is perhaps not surprising, then, that the field is characterized by a great many different specific models of growth. In contemporary economic theory, these models can be divided into two broad types: neoclassical growth theory (NGT) and heterodox growth theory (HGT).²

The purpose of this paper is to examine the structure of NGT and HGT with a view to establishing the possibilities for greater engagement between these traditions, and the challenges that would need to be surmounted if such engagement is to occur. A core assumption of the paper is that in any academic community charged with the creation and dissemination of knowledge, more (rather than less) interaction between researchers (and teachers) is always to be preferred. The benefits of greater engagement between NGT and HGT – and hence the motivation for this inquiry – follow directly from this assumption.

A central claim of the paper is that that there exists sufficient theoretical congruence between NGT and HGT to facilitate greater engagement. However, a mixture of methodological, rhetorical and sociological factors within the discipline of economics at large may thwart what could otherwise be profitable interaction between researchers in these different traditions.

The paper is organized as follows. Section 2 provides an overview of the core models that comprise NGT and HGT. All models are developed so as to emphasize the way they describe the basic “mechanics” of growth (Jones, 2002). The level of generality

¹ Growth was central to economic analysis for Classical economists such as Ricardo and Marx. In more recent times, Robert Solow was awarded the 1987 Nobel prize in economics for his contributions to the theory of economic growth.

² See, for example, Aghion and Howitt (2009) and Setterfield (2010) for comprehensive overviews of NGT and HGT, respectively.

so achieved enables us to describe the core insights of the NGT and HGT traditions in terms of just five structural models.³ Section 3 then discusses the key theoretical similarities between NGT and HGT that provide opportunities for greater engagement between these traditions, before section 4 examines methodological, rhetorical and sociological factors that may create barriers to such engagement. Finally, section 5 offers some conclusions.

2. Alternative Models of Growth

i) The Neoclassical Tradition

a) The Solow Model

Associated with the work of Solow (1956),⁴ the Solow model first emerged in response to the problems that Harrod (1939) identified as likely to encumber a growing economy. The model was subsequently treated as descriptive of the actual dynamics of a capitalist economy and, in retrospect, it can be thought of as the “first generation” of NGT.

The Solow model can be written as:

$$y_p \equiv q + n \quad [1]$$

$$q = \bar{q} \quad [2]$$

$$n = \bar{n} \quad [3]$$

$$y = y_p \quad [4]$$

³ A sixth structural model is introduced in section 3 to explain an important recent extension of the NGT tradition.

⁴ See also Swan (1956).

where y_p denotes the rate of growth of potential output (consistent with the full utilization of productive resources at any point in time), q is the rate of growth of labour productivity, n is the rate of growth of the labour force (which, assuming a constant rate of labour force participation in the long run, is equal to the rate of growth of the population) and y is the rate of growth of actual output. Equation [1] follows from the definition of the *level* of potential output as:

$$Y_p \equiv \frac{Y_p}{N_{\max}} \frac{N_{\max}}{L} \frac{L}{P} P$$

where $N_{\max}$ denotes the maximum level of employment associated, at any given point in time, with the (assumed constant) maximum rate of employment $N_{\max}/L$,⁵ L is the size of the labour force and P is the total population. Equations [2] and [3], meanwhile, treat the rates of growth of productivity and the labour force as exogenously given constants.

Finally, equation [4] is the “golden rule” for sustainable equilibrium growth in any steady-state growth model. Hence note that if we define $u = Y/Y_p$ as a simple measure of resource capacity utilization, where Y is the actual level of real output and Y_p is as previously defined, it follows that $\hat{u} = y - y_p$. Since u is bounded above and below, we must observe $\hat{u} = 0$ (i.e., $y = y_p$, as in [4]) in the steady state, in order for equilibrium growth to be sustainable in the long run. In neoclassical growth theory, however, equation [4] is also a causal statement, according to which y_p is understood to *determine* y .

⁵ The rate of employment $N_{\max}/L$ may be associated with full employment, or derived from either the natural rate of unemployment or the non-accelerating inflation rate of unemployment (NAIRU).

Solving the model in equations [1] – [4] yields the familiar exogenous growth result:

$$y = \bar{q} + \bar{n} \quad [5]$$

according to which the rate of growth of output is determined by the rates at which the productivity and availability of labour expand, both of which are taken as given.

b) Neoclassical Endogenous Growth Theory

Associated with, *inter alia*, Romer (1986, 1990) and Lucas (1988), neoclassical endogenous growth theory (NEGT) builds on the Solow model to create a “second generation” NGT, primarily by developing a theory of technical change.⁶

The essential claims of NEGТ can be represented by the system of equations:

$$y_p \equiv q + n \quad [1]$$

$$n = \bar{n} \quad [3]$$

$$y = y_p \quad [4]$$

$$q = q(X) \quad [6]$$

$$X = \bar{X} \quad [7]$$

where equation [6] is a technical progress function in which X is a vector of variables that affect the resources devoted to and/or the incentives to produce technological change.

The malleability of the precise specification of X and, indeed, of the precise functional form of [6] is what gives rise to the great variety of models associated with NEGТ.⁷

⁶ Indeed, Solow (1994, 2007) argues that its contributions to the economics of technical change are likely to prove to be the most enduring contribution of NEGТ.

⁷ See, for example, Aghion and Howitt (2009, pp.13-18) for a survey of the essential varieties of NEGТ. See Jones (2002, pp.164-6) for illustration of how the precise functional form of [6] can affect the results of NEGТ even as the vector X remains unchanged.

Solving the model in equations [1], [3], [4], [6] and [7] yields:

$$y = q(\bar{X}) + \bar{n} \quad [8]$$

according to which the rate of growth is endogenous in the sense that, given the rate of growth of the labour force, it is driven by technological change that: (a) is explicitly described by the technical progress function in [6]; and (b) occurs at a rate that is amenable to change in response to variation (by private decision makers and/or policy makers) in the vector X (Roberts and Setterfield, 2007, pp.14-16). Note, however, that if $X = n$, we have:

$$y = q(\bar{n}) + \bar{n} \quad [8a]$$

and the result is “semi-endogenous” growth (Jones 1995; 2002). On one hand, the rate of growth depends on the rate of technical progress as explicitly modelled in [6] (a feature that the result in [8a] shares with the basic NEG-T model summarized in [8]). However, the rate of technical progress and hence the rate of growth is no longer obviously amenable to change by either private or public decision makers within the economy, since both depend ultimately on the rate of growth of the population which is not a (narrowly-defined) economic variable. The resulting exogeneity of the growth rate is instantly recognizable as a distinguishing feature of the Solow model discussed earlier. The fact that the result in [8a] satisfies the first but not the second sense in which the rate of growth in [8] is endogenous – or in other words, that it hybridizes the results in [5] and [8] associated with the Solow model and NEG-T, respectively – is what gives “semi-endogenous” growth its name.

ii) *The Heterodox Tradition*

Various strands of HGT exist, the main demarcation being between the Classical and Keynesian traditions (see, for example, Marglin, 1984a).⁸ Rowthorn (1981) and Dutt (1984) mark the origin of the modern Kaleckian strand of the Keynesian literature, while Dixon and Thirlwall (1975) and Thirlwall (1979) began the modern Kaldorian strand. These strands, in turn, trace their origins to first-generation post-Keynesian growth theory in the work of Robinson (1956) and Kaldor (1966), respectively. The modern Classical tradition, meanwhile, can be traced back through Harris (1978) and Marglin (1984b) to Sweezy (1949).

a) The Canonical Heterodox Model

The basic tenets of HGT can be represented by a canonical model of the form:

$$y_p \equiv q + n \quad [1]$$

$$q = \bar{q} \quad [2]$$

$$n = \bar{n} \quad [3]$$

$$y = y(Z) \quad [9]$$

$$Z = \bar{Z} \quad [10]$$

where Z is a vector of variables that determines *either* the rate of growth of saving (Classical tradition) *or* the level and/or the rate of growth of autonomous demand (Keynesian tradition), and all other variables are as previously defined. Solving the model

⁸ Other strands of HGT exist that focus on evolutionary and/or unbalanced growth processes (see, for example, Cornwall, 1977; Pasinetti, 1981; Nell, 1990; Metcalfe and Foster, 2010). These are excluded from the discussion that follows, which focuses on steady-state growth models. See, however, Castellacci (2008) for comparative discussion of the trajectories of evolutionary growth theory and NEGT.

in equations [1] – [3] and [9] – [10] yields *two* growth rates. Hence as we have seen before, solving [1] – [3] yields:

$$y_p = \bar{q} + \bar{n} \quad [5]$$

while substituting [10] into [9] we get:

$$y = y(\bar{Z}) \quad [11]$$

Equations [5] and [11] represent the natural and the actual (equilibrium) rates of growth, respectively. The distinction between these two growth rates (per Harrod, 1939) reduces the natural rate of growth, y_p , to the status of a growth “ceiling” that sets an upper limit to the actual (equilibrium) rate of growth in the long run. Models of this genus then distinguish between two different types of growth regimes: one that is characteristic of labour constrained (or, following Robinson (1956), “golden age”) economies, where $y = y_p$ and growth conforms to the same pattern that would be observed in the neoclassical models outlined earlier; and one that is characteristic of non-labour constrained or “dual” economies (Skott and Ryoo, 2008), where $y = y(\bar{Z}) \neq y_p$ and the first Harrod problem (the inequality of the actual and natural rates of growth) is observed.

b) Endogenizing the Natural Rate of Growth

An important extension of the canonical HGT model described above involves treating the natural rate of growth in equation [5] as endogenous to the *actual* rate of growth, y (see, for example, León-Ledesma and Thirlwall, 2000, 2002; León-Ledesma and Lanzafame, 2010 for empirical evidence). For example, labour productivity growth can be described as a function of actual output growth by appeal to the Verdoorn law (on which, see McCombie et al, 2003). According to the Verdoorn law, the rate of technical

progress varies directly with the rate of growth – a dynamic analog of Adam Smith’s dictum that “the division of labour depends on the extent of the market”. This is because faster growth increases the pace of productivity-enhancing specialization within industries and firms, and their willingness to invest in “lumpy” capital that embodies technological improvements.⁹ In this case, the canonical HGT model outlined in the previous section must be re-written as:

$$y_p \equiv q + n \quad [1]$$

$$n = \bar{n} \quad [3]$$

$$y = y(Z) \quad [9]$$

$$Z = \bar{Z} \quad [10]$$

$$q = q(y) \quad [12]$$

where equation [12] – which replaces equation [2] – represents the Verdoorn law.

Solving the model again yields two different growth rates. Hence combining [9] and [10] we arrive, as before, at:

$$y = y(\bar{Z}) \quad [11]$$

But combining [1], [3] and [12] and bearing in mind the result in [11], we now find that:

$$y_p = q(y(\bar{Z})) + \bar{n} \quad [13]$$

Equations [11] and [13] once again distinguish between the actual and natural rates of growth, respectively. But rather than acting as an exogenously given growth ceiling, the natural rate in [13] now sets a maximum value of the growth rate at any point in time that is directly influenced by the equilibrium growth rate. This creates a form of path

⁹ The rate of growth may also affect the pace of innovation (see, for example, Schmookler, 1966) and, in dual economies, the evolution of the shares of employment in the traded goods and informal sectors and hence the rate of productivity growth (see, for example, Dutt and Ros, 2007).

dependence in the model, in the sense that the natural rate of growth will depend on the actual growth history of the economy, as captured by variations in the equilibrium rate of growth in [11] (see Setterfield, 2009, pp.42-4).¹⁰

c) Reconciling Actual and Potential Growth

According to the “golden rule” for sustainable, steady-state growth, only the labour-constrained or “golden age” variant of the canonical HGT model is strictly sustainable as a representation of equilibrium growth outcomes in the long run. Note that, even with an endogenous natural rate of growth, the model in the previous sub-section is not immune to this problem. Hence except in the special case where (from combination of [11] and [13]) we observe:

$$y(\bar{Z}) = q(y(\bar{Z})) + \bar{n}$$

we would expect the first Harrod problem ($y \neq y_p$) to arise, even when the natural rate is endogenous.

¹⁰ An alternative to the Verdoorn law approach used to derive the endogenous natural rate in [13] can be found in Palley (1996, 1997, 2002b) who, drawing on Kaldor (1957) and Scott (1989), specifies a technical progress function in which the rate of technical progress depends on the capital-labour ratio and the level of investment spending. Since investment is a variable that determines the level of autonomous demand in the Keynesian tradition – and is therefore an element of Z in this tradition – the Palley approach can be captured by a technical progress function of the form:

$$q = q(Z, \Omega) \quad [12a]$$

where Ω is a vector of other variables unrelated to the growth and/or level of autonomous demand. Combination of [1], [3], [10] and [12a] now yields:

$$y_p = q(\bar{Z}, \Omega) + \bar{n} \quad [13a]$$

The natural rate is once again endogenous, but this time because the demand-side determinants of the actual rate of growth also enter into the determination of productivity growth and hence the natural rate. Note, however, that as shown by Dixon and Thirlwall (1975, p.209), the Verdoorn law can be derived from Kaldor’s (1957) technical progress function, so Palley’s approach to modelling the endogenous natural rate can be encompassed by the Verdoorn law approach adopted in the text.

In view of all this, it is not surprising that there exists a literature in HGT that seeks to identify the mechanisms by which we might come to observe $y = y_p$ without abandoning the principle that the actual rate of growth is determined by the equilibrium solution of [9] and [10], as expressed in equation [11] (see, for example, Cornwall, 1972; Palley, 2002; Setterfield, 2006).¹¹ Models of this type begin with the proposition that the natural rate of growth is endogenous to the actual rate of growth, and are then structured so that the actual and natural rates of growth will be equalized (bearing in mind the endogeneity of the latter to the former) in equilibrium. A stylized representation of these models can be written as follows:

$$y_p \equiv q + n \quad [1]$$

$$n = \bar{n} \quad [3]$$

$$y = y(Z) \quad [9]$$

and *either*:

$$Z = \bar{Z} \quad [10]$$

$$q = q(y, u) \quad , \quad q_u > 0 \quad [14]$$

$$\dot{u} = u(y - y_p) \quad [15]$$

¹¹ The literature identified here is Keynesian in orientation. It is concerned with the possibility that the equilibrium rate of growth in [11] is demand-determined, and that it will remain so even in the long run. Otherwise, it would be straightforward to invoke the notion of a labour constrained economy as discussed earlier, and in so doing to accept that we must eventually accept $y = y_p$ as a *causal* statement. Coupled with the traditional interpretation of y_p as an exogenous natural rate of growth determined on the supply side, however, this would involve giving up the notion of demand-led growth.

Note also that the specific references above are to the Kaldorian tradition in demand-led growth theory. See, for example, Dutt (2006, 2009) for a parallel concern with reconciling the rates of growth of aggregate demand and aggregate supply within the Kaleckian tradition, in which the supply constraint on growth emanates (in the first instance) from the availability of capital and the target or “normal” rate of capacity utilization by firms, rather than the availability of labour. However, reconciliation of aggregate demand and aggregate supply in this tradition may still give rise to the first Harrod problem as in the canonical HGT model, and thus leave open the question of reconciling the actual and natural rates of growth that is explored above.

as in Setterfield (2006), *or*:

$$q = q(y) \quad [2]$$

$$\dot{u} = u(y - y_p) \quad [15]$$

$$Z = Z(\Upsilon, u) \quad , \quad Z_u < 0 \quad [16]$$

as in Palley (2002a), where u is the measure of resource capacity utilization introduced earlier, and equation [15] follows from the definition of u stated earlier. Equation [14] is a technical progress function based on an extended Verdoorn law, in which the rate of growth of productivity depends on both y and u . In Setterfield (2006, p.54), this is explained by the notion that the output elasticity of labour productivity (the so-called Verdoorn coefficient, $q_y > 0$) is increasing in u . In other words, the stimulus to q that emanates from any given rate of growth, y , via the Verdoorn law is higher the closer the economy operates to capacity, because firms will be more likely to engage in innovation and/or technical and organizational change when the goods market is *both* tight (so that firms are already operating close to capacity) *and* rapidly expanding. Equation [16], meanwhile, suggests that the proximate determinants of the actual rate of growth, Z , vary indirectly with u (and are also influenced by a vector of other variables, Υ). In Palley (2002a, p.121), this is explained by the notion that the income elasticity of imports is increasing in u . This is because higher values of u are associated with supply bottlenecks in domestic industry, which diverts demand abroad and thus lowers the demand-led rate of domestic output growth.

In the variant of the model in equations [1], [3], [9], [10], [14], and [15] (hereafter the “Setterfield model”), if $y > y_p$ then u will rise in equation [15], increasing q in [14]

which will, in turn, increase y_p in [1] towards the actual rate of growth which is determined by combination of [9] and [10]. This sequence of adjustments is captured in Figure 1, where $y(\bar{Z}) > y_p$ initially induces increases in the value of the natural rate associated with any given value of y , until the point $y(\bar{Z}) = y_p'$ is reached. Meanwhile, in the variant of the model in equations [1] – [3], [9], [15] and [16] (hereafter the “Palley model”), if $y > y_p$ then once again u will rise in equation [15]. This time, however, the rise in u will lower Z in [16] which will, in turn, decrease y in [9] towards the natural rate of growth, which is determined by combination of [1] – [3]. This sequence of adjustments is captured in Figure 2, where $y(\bar{Z}) > y_p$ initially induces reductions in the value of Z and hence y , until the point $y(\bar{Z}') = y_p'$ is reached.

Formally, equilibrium in either variant of the model is achieved when $y = y_p$ and hence $\dot{u} = 0$. Solving the Setterfield model under these conditions yields:

$$q(y(\bar{Z}), u^*) = y(\bar{Z}) - \bar{n} \quad [17]$$

where u^* is the equilibrium rate of resource capacity utilization. Notice that the “golden rule” for sustainable steady state growth is now satisfied in equilibrium, but in such a way that the natural rate of growth is the dependent variable (q and hence y_p adjusting to accommodate the value of $y = y(\bar{Z})$, as in Figure 1).

Solving the Palley model under the same equilibrium conditions yields:

$$y(Z(Y, u^*)) = q(y(Z(Y, u^*))) + \bar{n} \quad [18]$$

where, as before, u^* is the equilibrium rate of resource capacity utilization. Notice that the “golden rule” for sustainable steady state growth is again satisfied in equilibrium, but now in such a way that the actual rate of growth is the dependent variable (y adjusting to

accommodate the value of y_p in response to $y \neq y_p$ initially, as illustrated in Figure 2). In sum, in the solution to the Setterfield model, the natural rate of growth is the dependent variable in a model of “fully demand-determined growth,” in which adjustments on the supply side accommodate the demand-determined equilibrium rate of growth. In the solution to the Palley model, meanwhile, the actual rate of growth is the dependent variable in a model of “semi-supply-determined growth” – so called because, although it is the demand-determined equilibrium rate of growth that adjusts to accommodate the natural rate, the latter is directly affected by these adjustments on the demand side, as reflected in the reduction of the natural rate from y_p to y'_p in the movement towards sustainable steady state growth depicted in Figure 2. Note that the adjustment mechanisms postulated by Setterfield and Palley are not mutually exclusive. The possibility therefore exists that [14] and [16] are operative simultaneously, the result being a hybrid model in which adjustments on both the demand and supply sides of the economy play a role in determining the final steady state rate of growth consistent with the “golden rule” $y = y_p$.

3. Opportunities for Greater Engagement between NGT and HGT

Without doubt, there are a number of important theoretical differences between heterodox and neoclassical growth models. In the first place, the characterization of the supply side of the economy differs as between NGT and HGT. In neoclassical models, the supply side is understood in terms of technical relations of production between inputs and outputs. In heterodox models, meanwhile, these technical relations of production are

understood to be complemented by social relations of production between the owners of factor inputs – a legacy of the Classical economics that informs much HGT.

A second major difference between NGT and HGT concerns the role of the demand-side in determining growth outcomes – specifically, whether demand factors are ignored or taken seriously as potential determinants of long run growth. Hence the neoclassical models reviewed in section 2 are unambiguously supply-led. Both first-generation neoclassical growth theory (associated with the Solow model) and second-generation NEGТ are traditionally associated with the notion of supply-led growth, according to which the expansion of output in the long run is driven exclusively by increases in the productivity and/or availability of productive resources (principally capital and labour). The critical common feature of neoclassical growth theory of either generation, then, stems from the causal interpretation of equation [4] noted earlier, according to which y_p determines y and growth is a supply-led process. From this point of view, demand simply adjusts passively in the growth process, accommodating the expansion of potential output. This helps to explain the attitude of authors such as Stern (1991), who essentially *define* growth as a supply-led process.¹²

Keynesian variants of HGT, however, describe growth as a demand-led process: the proximate source of growth in these models is the level and/or the rate of growth of autonomous demand. Of course, not all HGT is Keynesian in inspiration, and HGT models that build on the Classical tradition typically characterize growth as a supply-led process. However, although growth is supply-determined in Classical theory, there exists

¹² “The study of growth is generally about the medium or long run. It is about the accumulation of physical capital, the progress of skills, ideas and innovation, the growth of population, how factors are used, combined and managed and so on. *It is therefore, principally, about the supply side*” (Stern, 1991, p.123. Emphasis added).

a well established tradition of taking seriously and debating the possibility of demand-led growth in Classical macroeconomics (see, for example, Dumenil and Levy, 1999). In this way, it can be said that the various strands of HGT share a common interest in properly locating the principle of effective demand in long run macrodynamics.

The dissimilarities identified above should not be allowed to conceal various common features of NGT and HGT, however. In the first place, since the object of analysis is identical for both NGT and HGT, it is not surprising to find that both traditions share certain analytical features. Hence as Gibson (2010) shows, *all* steady state growth theories satisfy a single equation of motion for the capital stock which can be derived from the fact that, by definition:

$$\frac{dK}{dt} \frac{1}{K} = \frac{I}{K} - \delta$$

where K denotes the capital stock, I is gross investment and δ is the rate of depreciation. From this perspective – as Gibson shows – individual NGT and HGT models are simply special cases of the same general claim about the dynamics of capital accumulation, the differences between them reducing to what they identify as the key exogenous “driver” of growth (for example, expansion of the labour force in the Solow model, or the “animal spirits” that govern investment spending in certain Keynesian growth models). Looked at through this lens, then, the “mechanics” of growth in both NGT and HGT are quite similar, even if the two traditions identify different factors as being the ultimate cause of economic expansion.

Second, an important common feature of HGT and second-generation NGT is that the equilibrium growth path is amenable to change (Palley, 2002b). In both traditions, the ultimate determinants of growth (the vectors X and Z) include elements that are

responsive to economic decisions and/or policy interventions (e.g., tax rates, or the distribution of income). From this perspective, a more compelling and important distinction than that between NGT and HGT is the distinction between models that give rise to solutions such as those found in [7] and [11], and models that give rise to solutions such as [8]. The latter are “fatalistic”: growth is a product of natural and/or engineering forces that are beyond economic control. The former, meanwhile, express a shared belief that public and/or private economic decision making can systematically alter the economy’s growth path.

Third, a major theoretical overlap between NGT and HGT now exists thanks to the discovery in NEGТ that trend and cycle may interact, with departures from trend associated with cyclical disturbances playing a critical role in the determination of the trend itself (see Gaggli and Steindl (2008) for a survey of this literature). Indeed, the mere fact that this theme has been broached by NGT provides a clear indication of convergence in the research agendas of NGT and HGT. Hence consider the following sentiments expressed by Kalecki over four decades ago:

the long-run trend is but a slowly changing component of a chain of short-period situations; it has no independent identity, and the two basic relations mentioned above [the multiplier and the accelerator] should be formulated in such a way as to yield the trend cum business-cycle phenomenon.

(Kalecki, 1968, p.263)

However, the similarities between NGT and HGT that result from consideration of the interaction of trend and cycle reach far beyond a shared vision. To see this, we begin by re-writing the stylized NEGТ model introduced earlier as:

$$y_p \equiv q + n \tag{1}$$

$$n = \bar{n} \tag{3}$$

$$q = q(X) \quad [6]$$

$$y = y_p + \varepsilon \quad [19]$$

$$\dot{X} = h(y - y_p) \quad , \quad h' \gtrless 0 \quad [20]$$

where $\varepsilon = \alpha\varepsilon_{-1} + \eta$ (with $0 < \alpha < 1$ and $\eta \sim (0, \sigma_\eta^2)$) describes a persistent shock to growth.¹³ As described in [19] (which replaces equation [4] in the “baseline” NEGТ model developed earlier), ε can separate the actual rate of growth from the potential rate of growth. Equation [20], meanwhile, replaces equation [7] in the baseline NEGТ model. It suggests that the supply-side determinants of technical progress are sensitive to the difference between y and y_p . Equation [20] is also consistent with the NEGТ hypothesis that aggregate fluctuations can have *either* a positive *or* a negative impact on the pace of growth, depending on whether the activities responsible for improving aggregate productivity (as captured by [6]) are compliments to or substitutes for the process of producing goods and services (Blackburn, 1999, p.68). Hence in one strand of NEGТ models (see, for example, Aghion and Howitt, 1992), recessions stimulate growth by “shaking out” inefficient practices (Schumpeter’s creative destruction) and by encouraging firms to devote more resources to innovation, so that $h' < 0$. In another strand, however (see, for example, Martin and Rogers, 1997), increased engagement in production stimulates learning by doing which has positive spillover effects on the productivity-enhancing activities modelled in [6] – so that $h' > 0$.¹⁴ It should also be

¹³ Note that if $\eta = \eta_1 \neq 0$ in some initial period and $\eta = 0$ thereafter, then the subsequent time path of ε is given by $\varepsilon_t = \alpha^{t-1} \eta_1$, so that $\lim_{t \rightarrow \infty} \varepsilon_t = 0$.

¹⁴ A simple example of the mechanisms operative in these different strands of the NEGТ literature can be developed by appealing to the Romer (1990) model of technical change, in which [6] takes the explicit form:

noted that the mechanisms emphasized above are not mutually exclusive. Hence there exists a class of NEG-T models in which $h' \begin{matrix} > \\ < \end{matrix} 0$, depending on whether “internal” learning (which results from explicitly devoting resources to learning and is counter-cyclical, as in creative destruction models) dominates ‘external’ learning (which results from learning by doing, and is pro-cyclical) (see, for example, Blackburn, 1999; Blackburn and Galindev, 2003; Galindev, 2009).

Under the equilibrium conditions $\eta = \varepsilon = 0$, we get $y = y_p$ from [19] and hence $\dot{X} = 0$ from [20]. Solving the model under these conditions yields:

$$y = q(X^*) + \bar{n} \quad [21]$$

a solution that is similar to that of the standard NEG-T model, in the sense that the potential rate of growth appears to determine the actual rate of growth. However, note that it is now the case that if $\eta \neq 0$ so that we observe $\varepsilon = \eta \neq 0$, the result will be $y \neq y_p$ in [19]. This will result in change in the value of X in [20], which will alter the value of q in [6] and hence y_p in [1]. This last sequence of events will, of course, alter the equilibrium value of the long run rate of growth in [21], to:

$$y = q(X') + \bar{n} \quad [21a]$$

$$q = q(X) = \kappa L_A$$

where L_A is research effort and κ is research productivity – specifically, the rate at which it is possible to transform the existing stock of knowledge into new ideas, per unit of research effort (see Jones, 2002, pp.101-106). hence in “creative destruction” models, $y < y_p$ will eliminate inefficient practices and stimulate innovation, thus increasing κ and L_A (respectively), and hence q . In “learning by doing” models, meanwhile, $y > y_p$ will stimulate learning by doing which will have spillover effects on the efficiency of research activity, thus increasing κ and hence q .

where $X' \neq X^*$. Hence despite the appearance of a conventional neoclassical result in [21], the potential rate of growth (and hence the long run actual rate of growth) is now sensitive to departures of the actual rate from its prior trend. Moreover, if ε captures the effect of persistent *demand* shocks (due, for example, to nominal rigidities in the goods market), the result is that the long run equilibrium rate of growth will be demand-determined, even as it satisfies the “golden rule” for sustainable steady state growth.¹⁵ This is, of course, exactly the result achieved in the class of Keynesian HGT model discussed in section 2(i)c.

The important point that emerges from the preceding analysis is that there exists a class of HGT models and an accompanying class of NEGТ models that yield observationally equivalent results: both posit that the long run equilibrium rate of growth is equivalent to the natural rate of growth, but that the latter is sensitive to the demand-determined actual rate of growth – with the result that the demand side of the economy now enters into the determination of long run growth outcomes. This result further substantiates the claim, advanced in this section, that there exist important examples of theoretical overlap between NGТ and HGT.

Indeed, the grounds for this claim can be extended further if, in the NEGТ model in equations [1], [3], [6], [19], and [20] above, we interpret $\varepsilon = \eta \neq 0$ as arising from demand shocks, and $h' > 0$ as the product of induced, factor biased technical change (see Foley and Michl, 1999, chpt.7; Duménil and Lévy, 1995, 2003). According to the latter process, when the actual rate of growth exceeds the natural rate so that employment growth exceeds the rate of growth of the labour force, the resulting fall in unemployment

¹⁵ This result is analogous to the effects that are observed in NAIRU or natural rate of unemployment models of the labour market in which hysteresis effects are postulated (see, for example, Cross, 1995).

will produce a “profit squeeze” (i.e., a fall in the profit share of income and hence, given the output-capital ratio, the rate of profit). This, in turn, induces firms to engage in technical change designed to displace labour and alleviate the profit squeeze. In so doing, the capital-labour ratio and hence (given the output-capital ratio) labour productivity increases. The result, then – as originally shown by Dutt (2006, 2010) – is a neoclassical model with a Classical technical progress function that produces Keynesian (demand-determined) long run growth outcomes. In short, the essential novelty of the NEG-T model in equations [1], [3], [6], [19] and [20] is that it connects the proximate “drivers” of technical change (X) to short run fluctuations. And this can be done in a variety of ways, including (as discussed above) “non-traditional” mechanisms borrowed from Classical economics. The result is a “grand synthesis” that combines Classical and neoclassical insights about the supply-side, but that ultimately allows for Keynesian (demand-determined) long run growth outcomes.

In sum, the discussion in this section suggests that there exists a considerable degree of theoretical overlap between neoclassical and heterodox growth theory. Of course, it could be argued that this claim involves undue emphasis on one branch of neoclassical growth theory (NEG-T in which shocks affect trend growth, *and* where the demand side is an important source of shocks) and that, even then, important differences exist between the precise causal mechanisms in observationally equivalent NGT and HGT models. In other words, the precise *theories* of growth differ as between these models, even if they do produce observationally equivalent results, so that “the devil is in the details”. But even allowing for these important caveats, there would seem to exist a basis for dialogue between proponents of NGT and HGT. Indeed, there even exists a

basis for recognizing that at the intersection of these traditions a common research agenda is emerging, focused on the interaction of trend and cycle and allowing for the possibility that variations in aggregate demand play a role in generating both short-run fluctuations and long-run growth.

4. Obstacles to Greater Engagement

Even if it is accepted that there exists significant theoretical overlap between NGT and HGT, obstacles remain to fruitful interaction between these traditions. Three different types of obstacles can be identified, associated with methodology, rhetoric and the sociology of the economics profession, respectively.

i) Methodological differences

Two important methodological differences between NGT and HGT are: i) the commitment of the former to providing “microfoundations” for macroeconomics, versus the commitment of the latter to aggregate structural modelling; and ii) the commitment of the former to an environment of full information and individual optimization, versus the commitment of the latter to an environment of fundamental uncertainty and norm-based behaviour.

These methodological differences have been extensively discussed and debated over the last four decades, and it is beyond the scope of this article to rehearse (much less resolve) them. The important point to note here is that they are anything but ephemera and, as such, they may fetter the opportunities for greater engagement among growth theorists identified in the previous section. Nevertheless, there are grounds for (cautious)

optimism. Hence as Solow's (1994, p.49; 2007, pp.7-8) remarks on the historical development of NGT make clear (with reference to the Sonnenschein-Mantel-Debreu (SDM) theorems in general equilibrium theory), the representative agent approach to modelling currently favoured by NGT provides no advance over aggregate structural models when it comes to providing a "microfoundation" for macroeconomics. This internal criticism of contemporary NGT suggests that, based on observation of its current modelling methods, the first of the methodological differences identified above *should* not be used as a basis for denying the possibility of dialogue between NGT and HGT.

Moreover, drawing on the self-same SDM results, some researchers have advocated and begun to develop a "second generation" microfoundations project that situates heterogeneous agency at its core (see, for example, Kirman, 1989, 1992). This project is antipathetic to the "first generation" microfoundations project, and not only because it explicitly denies the homogeneity of individual agency on which rests the very claim of the first generation project to provide a microfoundation for macroeconomics. It is also anti-reductionist, in the specific sense that it denies the possibility of any simple discussion of wholes in terms of the properties of their constituent parts, by virtue of the existence of *emergent properties*: features of a system at one level of aggregation that are not evident at, and cannot be explained in terms of the characteristics of whatever exists at, lower levels of aggregation. The existence of emergent properties has long been a concern of aggregate structural modelling in macroeconomics, as evidence by its concern with fallacies of composition – from Keynes's paradox of thrift (which suggests that an increase in the propensity to save of all individual households will leave the aggregate volume of saving unchanged) to Harrod's instability problem (which arises from firms

antagonizing the problem of aggregate excess capacity by cutting back on investment in an effort to reduce excess capacity). These observations suggest that if an increasing number of macroeconomists come to share a common vision of their object of analysis (characterized by emphasis on emergent properties) and hence a common methodological stance (characterized by anti-reductionism), then some of the methodological tensions that *may* obstruct dialogue between NGT and HGT in principle *will not do so* in practice.

ii) Getting a word in edgeways: the rhetoric of growth theory

Scholarly exchange, like any conversation, requires a common language and at least some common understanding of what has been discovered to date (“what we know”) and what, therefore, constitutes the agenda for further research (“what is to be done”). It can be argued that growth theory, as a field, lacks both to some degree. Hence Setterfield (2002, 2003) laments the neoclassical “capture” of the *history* of growth theory. According to this view, it has become commonplace to regard growth theory as having begun with the development of the Solow model and then, following a brief hiatus during the 1970s and early 1980s, re-commenced with the introduction of NEGТ. This overlooks both the Classical contributions and even those of Harrod prior to the work of Solow (1956), and the many contributions that continued the development of HGT during the so-called hiatus in growth theory after 1970.¹⁶ Meanwhile, Roberts and Setterfield (2007) criticize the neoclassical capture of the *language* of growth theory, which has seen the term “endogenous growth” become synonymous with NEGТ. This despite the fact that HGT models have traditionally produced endogenous growth – either in the “narrow” sense that technical change is explicitly modelled and/or the growth rate varies

¹⁶ These include all of the “founding” contributions to modern HGT identified in section 2(ii).

with the equilibrium solution of the model (which is, in turn, amenable to change by economic decision makers), or in the more expansive sense that growth depends on its own past history (and is therefore path dependent).

The problems identified above are real and therefore important in practice. But in and of themselves they lack intellectual substance: they *should* not be allowed to fetter conversation among growth theorists inspired by the theoretical overlap between NGT and HGT. Whether or not they *will*, of course, remains to be seen.

iii) It takes two to tango (and to have a conversation): the sociology (and economics) of economics

A third potential obstacle to greater engagement between NGT and HGT is an unwillingness to converse. *Prima facie* evidence suggests that this problem is real. Hence while HGT is replete with references to neoclassical growth literature (see, for example, Setterfield (2002, 2003), Dutt (2006, 2009), and Leon-Ledesma and Lanzafame (2010) on NEGТ models in which demand matters), this cross-referencing is asymmetric. For example, in their seminal contributions to the NEGТ literature in which trend and cycle interact, neither Aghion and Howitt (1992) nor Martin and Rogers (1997) make any reference to the HGT literature in which, as the quotation from Kalecki in section 3 illustrates, the interaction of trend and cycle has long been understood to be important.¹⁷

It is not difficult to account for this asymmetry in terms of the social structure of the economics profession (on which see, for example, Hands, 1994). First, networks of

¹⁷ To their credit, Gaggli and Steindl (2008) *do* refer to Kalecki in their survey of the NEGТ literature in which trend and cycle interact, but only in a footnote designed to establish that “there has not been much emphasis – at least within mainstream macroeconomic research – on the interrelation of trend growth and business cycles until very recently” (Gaggli and Steindl, 2008. p.1).

“gurus and groupies” exist within both NGT and HGT. The internal dynamics of these networks – as with those of other social networks – are self-reinforcing: the network identity of any given individual is increasing in the individual’s exposure to other members of the network. Meanwhile, their relative size alone suggests that even if researchers interacted randomly, it is far more likely that an NGT researcher will encounter another NGT researcher and will thus expand his/her frame of reference only *within* the network to which he/she already belongs. Second, there are varying levels of commitment to pluralism as an educational first principle within academic economics. This will affect the likelihood that any given researcher will deliberately incorporate dissenting views into his/her teaching (which may, in turn, affect his/her research) out of a sense of educational duty to his/her students. Finally, these sociological factors are reinforced by a potentially important facet of the “economics of economics”. Simply put, it is not in the self-interest of a majority (such as NGT researchers) to deliberately seek out dissent if this only serves to increase the costs of their own intellectual reproduction, in much the same way that organisms bent on physical reproduction will not deliberately seek out rivals for their mates.

The structural impediments to engagement described here may well prove formidable. Hence while it can be argued that “ignorance is not an excuse” and that an unwillingness to converse *should* not impede scholarly engagement, there is reason to believe it *will* in practice.

5. Conclusion

This paper has explored the basic structures of both neoclassical and heterodox growth theories, with a view to investigating both the possibilities for and the likely impediments to greater engagement between researchers in these traditions. A number of simple structural models have been developed that describe the essential “mechanics” of growth in both NGT and HGT. Interpretation of these models suggests that there exists considerable theoretical overlap – and even cases of observational equivalence – as between the NGT and HGT traditions.

In and of itself, this ought to provide a basis for increased engagement between researchers associated with these traditions. Whether or not this actually occurs, however, will depend on the willingness and ability of growth theorists to overcome a number of obstacles to their interaction, variously associated with the methodology, rhetoric and sociology of economics. A central premise of this article is that in any academic community, interaction between researchers is always valuable. What is less easily discerned is whether, in the case of growth theory, such interaction is deemed sufficiently valuable to motivate researchers from different theoretical traditions to actively confront impediments to their greater engagement.

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Figure 1: Adjustment towards sustainable steady-state equilibrium in the Setterfield model

Figure 2: Adjustment towards sustainable steady-state equilibrium in the Palley model